A lattice perspective of kaon phenomenology[*]

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Abstract. I review recent lattice computations of the matrix element relevant for $K^0 - \overline{K}^0$ mixing and discuss the advantages of fermions with an exact chiral symmetry to compute $K \to \pi\pi$ amplitudes.

PACS. 11.30.Er Discrete symmetries – 13.25.Es Decays of K mesons – 12.38.Gc Lattice QCD calculations

1 Introduction

Kaon decays are sources of key informations to test the flavour structure of the Standard Model and to search for new physics. Recently the experimental results for indirect CP violation in $K \to \pi\pi$ decays and those from B decays and oscillations were compared to test the Cabibbo–Kobayashi–Maskawa picture of flavour mixing and CP violation [1,2,3]. In these analyses the experimental result for ε is connected to the fundamental parameters of the underlying electroweak theory through the lattice determination of the relevant hadronic matrix element of the $\Delta S = 2$ effective Hamiltonian. Its bag parameter $\hat{B}_K$ has been computed in the quenched approximation over the past decade with a remarkable accuracy. A summary estimate of $\hat{B}_K$ is given in the first part of this talk, followed by a review of recent computations.

Quantitative tests of the same quality have not been possible so far in the $\Delta S = 1$ sector for the so-called $\Delta I = 1/2$ rule in $K \to \pi\pi$ decays or for the direct CP violation parameter ε'/ε. In this sector a more complicated blend of ultraviolet and infrared effects renders the determination of the relevant matrix elements very challenging and prevented so far reliable computations with standard regularizations. In the ultraviolet, power-divergent subtractions can be needed to construct the renormalized operators that enter the effective Hamiltonian [4,5,6,7]. In the infrared, the continuation of the theory to Euclidean space-time and the use of finite volumes in numerical simulations generate a non-simple relation between the physical amplitudes and those computed on the lattice [8,9,10]. The use of lattice fermion discretizations which preserve an exact chiral symmetry at finite lattice spacing greatly simplifies some of these computations and allows one to attack them as shown in the second part of this talk[1].

Before entering into details, it is important to stress that lattice QCD allows one to perform non-perturbative computations of masses and matrix elements from first principles, with control, at least in principle, over all sources of systematic errors. These uncertainties can be systematically reduced by exploiting the properties of the underlying quantum field theory and/or by using more powerful computers, but without the introduction of any model-dependent assumption or parameter. Examples are given by the $O(a)$-improved actions and operators and by the non-perturbative techniques used to renormalize bare matrix elements.

Up to now many computations are still performed in the so-called quenched approximation, i.e. dropping the vacuum polarization effects in the Monte Carlo simulations. For some of them this represents the only source of systematic uncertainty not under control.

2 K→ ππ decays

Non-leptonic $K \to \pi\pi$ amplitudes can be parametrized as

$$T[K^+ \to \pi^+\pi^0] = \sqrt{\frac{3}{2}} A_2 e^{i\delta_2} \qquad (1)$$

$$T[K^0 \to \pi^+\pi^-] = \sqrt{\frac{2}{3}} A_0 e^{i\delta_0} + \sqrt{\frac{1}{3}} A_2 e^{i\delta_2}$$

$$T[K^0 \to \pi^0\pi^0] = \sqrt{\frac{2}{3}} A_0 e^{i\delta_0} - 2\sqrt{\frac{1}{3}} A_2 e^{i\delta_2},$$

where δ_I and A_I are the $\pi\pi$ phase shifts and the isospin amplitudes for $I = 0, 2$. Direct and indirect CP violations

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[1] For lack of space only a selection of topics are reviewed here, chosen following my personal taste and competence. Please see [11] for an up-to-date report on rare kaon decays and [12] for a broader overview on the lattice activity in kaon physics.

are parametrized by

$$\varepsilon' = \frac{1}{\sqrt{2}} e^{i\Phi} \frac{\mathrm{Re}A_2}{\mathrm{Re}A_0} \left(\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0} \right) \qquad (2)$$

and

$$\varepsilon = \frac{T[K_L \rightarrow (\pi\pi)_0]}{T[K_S \rightarrow (\pi\pi)_0]} \qquad (3)$$

respectively. Experimental results reveal $\Phi = \pi/2 + \delta_2 - \delta_0 \approx \pi/4$, a $\Delta I = 1/2$ selection rule $|A_0/A_2| \simeq 22.2$, the presence of direct and indirect CP violation in nature

$$\mathrm{Re}(\varepsilon'/\varepsilon) = (16.6 \pm 1.6) \times 10^{-4} \qquad [13,14]$$
$$|\varepsilon| \;\; = (2.282 \pm 0.017) \times 10^{-3} \; [15] \,. \qquad (4)$$

The large value of $|A_0/A_2|$ and that of ε'/ε can be explained within the Standard Model only if the strong interactions crucially affect these non-leptonic weak transitions (see [16,17,18] for recent reviews).

2.1 K^0–$\bar{K}^0$ mixing: ε

By using the OPE, the $\Delta S = 2$ effective Hamiltonian is given by

$$H_{\mathrm{eff}}^{\Delta S=2} = \frac{G_F^2 M_W^2}{4\pi^2} \mathcal{C}_1(\mu) \widehat{\mathcal{O}}_1(\mu) + \mathrm{h.c.} \,, \qquad (5)$$

where the expression of the Wilson coefficient $\mathcal{C}_1(\mu)$ as a function of the fundamental parameters of the underlying theory is known at NLO [19] and the corresponding bare four-fermion operator is given by

$$\mathcal{O}_1 = (\bar{s}\gamma_\mu P_- d)(\bar{s}\gamma_\mu P_- d)\,, \qquad P_\pm = \frac{1 \pm \gamma_5}{2} \,. \qquad (6)$$

Its matrix element between $\overline{K}^0$ and K^0

$$\langle \overline{K}^0 | \widehat{\mathcal{O}}_1 | K^0 \rangle \equiv \frac{4}{3} F_K^2 M_K^2 \widehat{B}_K \qquad (7)$$

encodes the long-distance QCD contributions to ε. So far the most precise and careful computation of $\widehat{B}_K(\mu)$ in the quenched approximation has been carried out by JLQCD [20] (see also [21]). They used staggered fermions, one-loop perturbation theory to renormalize the operator and degenerate down and strange quark masses. Their results at fixed lattice spacings are reported in Fig. 1 together with their continuum extrapolated value

$$\widehat{B}_K^{\overline{\mathrm{MS}}}(2\mathrm{GeV}) = 0.63 \pm 0.04 \qquad (8)$$
$$\widehat{B}_K = 0.86 \pm 0.06 \,, \qquad (9)$$

where $\widehat{B}_K$ is the renormalization group-invariant bag parameter. The final error of 0.04 is much larger than the statistical ones at fixed lattice spacing. The amplification is due to the fact that with these fermions $O(a^2)$ effects in the matrix element are found to be large, and a consistent continuum limit of the results can be obtained only

if also $O(\alpha_s^2)$ corrections are allowed in the extrapolation fit. By using chiral perturbation theory, Sharpe estimates a 5% systematics due to the degeneracy of the quark masses [22]. At present the systematic error due to the quenching approximation is not under control. A crude estimate based on quenched chiral perturbation theory and partially quenched simulations suggests to include a further 15% error in (8) [22]. The error on $\widehat{B}_K$ can be put under control and significantly reduced only with full QCD simulations with realistic light-quark masses. Despite a great effort in the last few years in the lattice community [23,24], the simulation of dynamical quarks is still very challenging and a breakthrough in numerical algorithms may still be needed to reach interesting light-quark masses.

In the last year several collaborations have been computing $\widehat{B}_K$ in quenched QCD by using sophisticated regularizations, non-perturbative renormalization techniques and allowing for $m_s \neq m_d$ [25,26,27]. The aim is to check the staggered result and to pin down the systematic error within the quenched approximation. Even if a significant comparison can be performed only when all systematics (within the quenching approximation) will be under control also in these computations, it is interesting to analyse and compare the results to appreciate their potentiality.

Two collaborations [25,26] computed $\widehat{B}_K$ with Ginsparg–Wilson fermions [28,29,30,31] for the first time. These regularizations simultaneously preserve chiral and flavour symmetries at finite lattice spacing [32]. As a consequence $\mathcal{O}_1$ renormalizes multiplicatively and its matrix elements are $O(a)$-improved. The plain Neuberger action [30] has been used in [25] for a lattice of linear extension $L \simeq 1.5$ fm, a spacing $a \simeq 0.09$ fm and for degenerate light-quark masses. The RI/MOM non-perturbative renormalization procedure has been implemented to compute the logarithmic divergent renormalization constant. The result obtained is reported in Fig. 1. Even with a large error, it is in very good agreement with the continuum extrapolated staggered result, suggesting that also for this matrix element discretization effects may be mild with Neuberger's fermions. A NNC–HYP overlap fermion action has been implemented in [26] for lattices of linear extension $L \simeq 1.3$ fm, $a \simeq 0.08, 0.12$ fm and for degenerate light-quark masses. The results are lower but still compatible within two sigmas with the continuum extrapolated staggered one and are in the same ballpark as those obtained with domain-wall fermions with a finite fifth dimension [33,34]. Even if both studies require more work to properly assess the magnitude of the various systematic errors, they confirmed that Ginsparg–Wilson fermions can be very effective in computing phenomenological quantities despite their numerical cost.

In the last few years the ALPHA collaboration has been computing $\widehat{B}_K$ with the goal of reaching a very precise determination of $\widehat{B}_K$ within the quenched approximation. They are using twisted-mass fermions, which break flavour symmetry and violate parity but allow for a multiplicatively renormalization of $\mathcal{O}_1$. A non-perturbative determination of the renormalization constant and its running
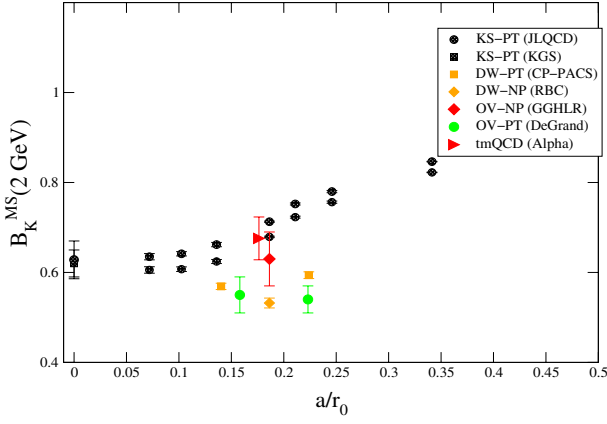
Fig. 1. $B_K^{\overline{\text{MS}}}(2\,\text{GeV})$ versus the lattice spacing in units of the Sommer scale r_0 [35] from the reference computation, black circles [20] (see also also black squares [21]), and from recent determinations: red diamonds [25], green circles [26], red triangle [27], orange diamonds [33], orange squares [34]

has been completed, while the computation of the matrix element is still under way. A preliminary result has been reported in [27] and is shown in Fig. 1. Also in this case the result obtained at $a \simeq 0.09$ is in very good agreement with the continuum extrapolated staggered result.

2.2 The $\Delta I = 1/2$ rule

By using the operator product expansion (OPE), the CP-conserving $\Delta S = 1$ effective Hamiltonian above the charm threshold is given by [36,37,38]

$$H_{\text{eff}}^{\Delta S=1} = \frac{G_F}{\sqrt{2}}\Big[C_+(\mu)\widehat{\mathcal{O}}_+(\mu) + C_-(\mu)\widehat{\mathcal{O}}_-(\mu)\Big],$$

where the Wilson coefficients $C_\pm(\mu)$ are known at the NLO [39,40] and the bare operators are

$$\mathcal{O}_\pm = \Big[(\bar{s}^a\gamma_\mu P_- u^b)(\bar{u}^b\gamma_\mu P_- d^a) \tag{10}$$
$$\pm (\bar{s}\gamma_\mu P_- u)(\bar{u}\gamma_\mu P_- d)\Big] - (u \to c) .$$

The contributions that arise when the top quark is integrated out are heavily suppressed by CKM factors and can be neglected. $\mathcal{O}_\pm$ belong to different chiral multiplets and are CPS-even. In correlation functions at non-zero physical distance, $\mathcal{O}_\pm$ cannot mix between themselves or with other four-fermion operators if the regularization preserves chiral symmetry, but only with the dimension-six operator [5,7]

$$\mathcal{Q}_m = (m_u^2 - m_c^2)\Big[m_d(\bar{s}P_+ d) + m_s(\bar{s}P_- d)\Big] . \tag{11}$$

The renormalized operators are

$$\widehat{\mathcal{O}}_\pm(\mu) = Z_\pm(\mu)\Big[\mathcal{O}_\pm + b_\pm^m \mathcal{Q}_m\Big], \tag{12}$$

where $Z_\pm(\mu)$ are logarithmic-divergent renormalization constants and $b_\pm^m$ are suppressed by a factor α_s. No

power-divergent subtractions are needed to renormalize $\mathcal{O}_\pm$ when fermions with an exact chiral symmetry are used [7].

For $m_s \neq m_d$,

$$\mathcal{Q}_m = (m_u^2 - m_c^2)\partial_\mu\Big[\frac{m_d + m_s}{m_s - m_d}\mathcal{V}_\mu^{sd} \tag{13}$$
$$+ \frac{m_d - m_s}{m_s + m_d}\mathcal{A}_\mu^{sd}\Big]$$

and it does not contribute to matrix elements that preserve four-momentum [4,5].

If the charm is integrated out, not only potentially large contributions of $O(\mu^2/m_c^2)$ are neglected, but ultraviolet power divergences can arise in the renormalization pattern of the relevant four-fermion operators. In this case the $\Delta S = 1$ effective Hamiltonian can be written as

$$H_{\text{eff}}^{\Delta S=1} = \frac{G_F}{\sqrt{2}}\sum_{i=1}^{10} C_i(\mu)\widehat{\mathcal{Q}}_i(\mu) . \tag{14}$$

The so-called QCD-penguin operators are

$$\mathcal{Q}_{3,5} = (\bar{s}\gamma_\mu P_- d)\sum_{q=u,d,s}(\bar{q}\gamma_\mu P_\mp q) \tag{15}$$

$$\mathcal{Q}_{4,6} = (\bar{s}^a\gamma_\mu P_- d^b)\sum_{q=u,d,s}(\bar{q}^b\gamma_\mu P_\mp q^a) \tag{16}$$

(see [16,17] for definitions of the other operators). At non-zero physical distance, mixing with two lower-dimensional operators

$$\mathcal{Q}_p = m_d(\bar{s}P_+ d) + m_s(\bar{s}P_- d) \tag{17}$$
$$\mathcal{Q}_\sigma = m_d(\bar{s}F_{\mu\nu}\sigma_{\mu\nu}P_+ d) + m_s(\bar{s}F_{\mu\nu}\sigma_{\mu\nu}P_- d)$$

can occur and power-divergent subtractions are needed even with Ginsparg–Wilson fermions.

With Wilson fermions, only CPS and flavour symmetry can be used to determine the renormalization pattern of $\mathcal{O}_\pm$. Even with an active charm, a quadratic divergent contribution needs to be subtracted in the parity-conserving sector

$$\widehat{\mathcal{O}}_\pm^{\text{PC}}(\mu) = \mathcal{Z}_\pm(\mu)\Big[\mathcal{O}_\pm^{\text{PC}} + \sum_j b_\pm^j \mathcal{O}_\pm^j +$$
$$+ b_\pm^\tau \mathcal{Q}_\tau + \frac{b_\pm^s}{a^2}\mathcal{Q}_s\Big], \tag{18}$$

where

$$\mathcal{Q}_s = (m_u - m_c)\bar{s}d \tag{19}$$
$$\mathcal{Q}_\tau = (m_u - m_c)\bar{s}\,\sigma_{\mu\nu}F_{\mu\nu}d . \tag{20}$$

and $\mathcal{O}_\pm^j$ are four-fermion operators with wrong chirality. In this case only the flavour part of the GIM mechanism survives thanks to the explicit breaking of chiral symmetry [5]. It is interesting to notice that power divergences can be mitigated with twisted-mass Wilson fermions [41].

At leading order in chiral perturbation theory, the matrix elements needed for the $\Delta I = 1/2$ rule can be extracted from three-point correlation functions, thus avoiding the infrared problem that affects the direct computation of the $K \to \pi\pi$ matrix elements on the lattice [4].

No large cancellations among leading order terms are expected in the ratio $|A_0/A_2|$; an enhancement should therefore be visible already at this order. As a result, a combined use of fermions with an exact chiral symmetry and chiral perturbation theory can be the starting point to attack the $\Delta I = 1/2$ rule.

In the past years the RBC and the CP-PACS collaborations have studied the $\Delta I = 1/2$ rule[2] with domain-wall fermions with a finite fifth dimension [33,42]. They have computed the $K \to \pi$ and $K \to 0$ matrix elements for the operators of the $\Delta S = 1$ effective Hamiltonian in (14) and used LO chiral perturbation theory to recover the physical amplitudes. Albeit with large statistical and systematic errors, both groups demonstrated that a controlled numerical signal can be obtained for these matrix elements. The systematics of these important results can be put under control by using fermions with an exact chiral symmetry, lighter quark masses, and by considering the effective Hamiltonian with a dynamical charm. A numerical feasibility study with Neuberger's fermions is under way [43]. It is also conceivable that the relevant low-energy constants of the weak chiral Lagrangian can be extracted by studying the weak interactions in the ϵ-regime [43,44,45,46].

For direct CP violation, both the ultraviolet and the infrared problems are more severe. An active charm does not mitigate the ultraviolet renormalization, and divergent power subtractions are necessary. The cancellation between two large competing contributions from $\mathcal{Q}_6$ and $\mathcal{Q}_8$ renders ε'/ε very sensitive to higher order corrections in chiral perturbation theory. Leading-order terms may not be sufficient to reach a reliable prediction in the Standard Model [47,48].

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[2] See [49] for an earlier attempt with staggered fermions.